# 2203000205023003
# EXAMINATION FEBRUARY-MARCH 2024
# BACHELOR OF SCIENCE (FIFTH SEMESTER)
# MATHEMATICS-XIII (MTH-503-REAL ANALYSIS-I)
# LEVEL 2

[**Time: As Per Schedule]** **[Max. Marks: 50]**

**Instructions:**
**1. Fill up strictly the following details on your answer book**
  a. Name of the Examination : **BACHELOR OF SCIENCE (FIFTH SEMESTER)**
  b. Name of the Subject : **MATHEMATICS-XIII (MTH-503-REAL ANALYSIS-I) LEVEL 2**
  c. Subject Code No : **2203000205023003**
2. Sketch neat and labelled diagram wherever necessary.
3. Figures to the right indicate full marks of the question.
4. All questions are compulsory.
5. Follow usual notations.

Seat No:

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Student's Signature

**Q.1 Attempt any five:(2 marks each)** **10**

1. Define bounded Sequence. Is $\{(-1)^n\}_{n=1}^{\infty}$ is bounded? Justify your answer.
2. Show that the sequence $\left\{\frac{1}{1+n^2}\right\}_{n=1}^{\infty}$ is monotone.
3. Give an example of sequences $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ for which as n→ ∞ :
$$s_n \to \infty, t_n \to \infty \; and \; s_n - t_n \to 7.$$
4. Find the limit superior and the limit inferior of the sequence 1, 2, 3, 1, 2, 3, 1, 2, 3, ...
5. Define: Convergence of the series of real numbers $\sum_{n=1}^{\infty} a_n$
6. Show that the series $\sum_{n=1}^{\infty} \frac{1-n}{1+2n}$ diverges
7. Prove that if $a_1 + a_2 + a_3 + \ldots$ converges to s then $a_2 + a_3 + a_4 + - - -$ converges to $s - a_1$

8. Give an example of a series which converges conditionally.

**Q.2 Attempt any two:(5 marks each) 10**

1. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent then prove that $\{s_n\}_{n=1}^{\infty}$ is bounded.

2. For $n \in I$ $if$ $s_n = \frac{2.4.6...2n}{1.3.5...(2n-1)} \frac{1}{n^2}$ then prove that $\{s_n\}_{n=1}^{\infty}$ is nonincreasing.

3. If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, if $c \in R$, and if $\lim_{n\to\infty} s_n = L$ then show that $\lim_{n\to\infty} cs_n = cL$ Also show that $\lim_{n\to\infty} \frac{2n^3+5n}{4n^3+n^2} = \frac{1}{2}$

**Q.3 Attempt any two:(5 marks each) 10**

1. If $\{s_n\}_{n=1}^{\infty}$diverges to infinity and if $\{t_n\}_{n=1}^{\infty}$ converges then prove that $\{s_n + t_n\}_{n=1}^{\infty}$ diverges to infinity.

2. Find the limit superior and the limit inferior of the sequence $\{\sin(\frac{n\pi}{2})\}_{n=1}^{\infty}$.

3. If the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges then show that $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

**Q.4 Attempt any two:(5 marks each) 10**

1. If $\sum_{n=1}^{\infty} a_n$ is a convergent series then show that $\lim_{n\to\infty} a_n$ = 0. Is converse true. Justify your answer.

2. Prove that the series $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \cdots$ converges.

3. Prove that the series $(1 - 2) - \left(1 - 2^{\frac{1}{2}}\right) + \left(1 - 2^{\frac{1}{3}}\right) - \left(1 - 2^{\frac{1}{4}}\right) +..$.Converges

**Q.5 Attempt any two:(5 marks each) 10**

1. If $\sum_{n=1}^{\infty} b_n$ converges absolutely and if $\lim_{n\to\infty} \frac{|a_n|}{|b_n|}$ exists then show that $\sum_{n=1}^{\infty} a_n$ converges absolutely.
2. Show that the series $\sum_{n=1}^{\infty} \frac{1}{2n+5}$ diverges
3. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

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